Computer Graphics

Chapter_4

2D Transformations

Lecture #2

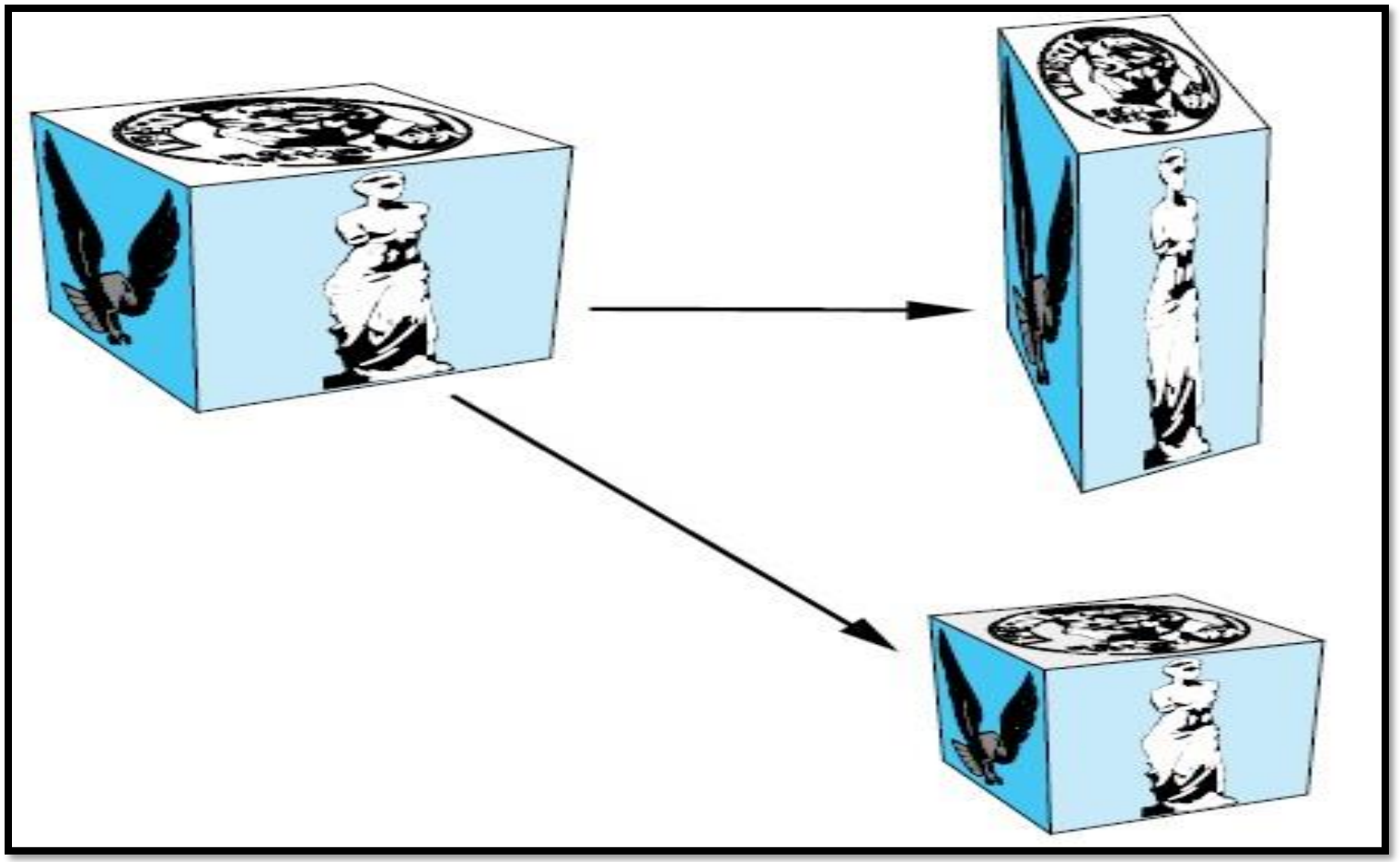
Scaling

- A scaling transformation alters the size of an object.
- This operation can be carried out for polygons by multiplying the coordinate values (x, y) of each vertex by scaling factors s_x , and $\boldsymbol{s_y}$, to produce the transformed coordinates (x', y'):

$$x' = s_x \,.\, \boldsymbol{x}$$

$$y' = s_y \,.\, \boldsymbol{y}$$

- s_x scales objects in the **x** direction.
- s_y scales objects in the **y** direction.



Uniform and non-uniform scaling

- Scaling is a transformation by which we can make an object **bigger** or **smaller**.
- Scaling makes an object **bigger** if the scaling factors are greater than **"1"**. **This case is called "Magnification"**.
- Scaling makes an object **smaller** if the scaling factors are smaller than **"1"**. **This case is called "Reduction"**.
- Scaling is called uniform if:

$$s_x = s_y$$

- Scaling is called non-uniform if:

$$s_x \neq s_y$$

- Scaling factors s_x, and s_y are non-negatives. Any positive numeric values can be assigned to the scaling factors.

- Specifying a value of 1 for both s_x, and s_y, leaves the size of objects **unchanged**.

- Uniform scaling of a circle is done by simply adjusting the radius. Then we redisplay the circle about the center coordinates using the transformed radius.
- An ellipse in standard position is resized by scaling the semi-major and semi-minor axes and redrawing the ellipse about the designated center coordinates.

- Expressing positions in homogeneous coordinates allows us to represent all geometric transformation equations as matrix multiplications.
- Coordinates are represented with three-element column vectors, and transformation operations are written as 3×3 matrices.

- For Scaling. we have: $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s_x} & 0 & \mathbf{0} \\ 0 & \boldsymbol{s_y} & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

- The Scaling matrix can be written in the abbreviated form:

$$\boldsymbol{P' = S(s_x, s_y).\ P}$$

with $\boldsymbol{S(s_x, s_y)}$ as the 3 by 3 translation matrix.

- The inverse of the Saling matrix is obtained by replacing the scaling factors $\boldsymbol{s_x}$ and $\boldsymbol{s_y}$ with their inverses: $\frac{\boldsymbol{1}}{\boldsymbol{s_x}}$ and $\frac{\boldsymbol{1}}{\boldsymbol{s_x}}$.

Composite Transformations

- Composite transformation represent any sequence of transformations.
- With the matrix representations we can set up a **matrix** for any sequence of transformations as a composite transformation matrix by calculating the matrix product of the individual transformations.
- **First kind**: Two successive transformations of the same kind.
- **Second kind**: Successive transformations of different kinds.

- **Two successive translations is:**

$$\begin{bmatrix} 1 & 0 & t_{x1} \\ 0 & 1 & t_{y1} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_{x2} \\ 0 & 1 & t_{y2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{x1} + t_{x2} \\ 0 & 1 & t_{y1} + t_{y2} \\ 0 & 0 & 1 \end{bmatrix}$$

- **Two successive scaling is:**

$$\begin{bmatrix} s_{x1} & 0 & 0 \\ 0 & s_{y1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_{x2} & 0 & 0 \\ 0 & s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{x1} . s_{x2} & 0 & 0 \\ 0 & s_{y1} . s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- **Two successive rotation is:**

$$\begin{bmatrix} \cos\theta & -\sin\theta & \mathbf{0} \\ \sin\theta & \cos\theta & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta & \mathbf{0} \\ \sin\beta & \cos\beta & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+\beta) & -\sin(\theta+\beta) & \mathbf{0} \\ \sin(\theta+\beta) & \cos(\theta+\beta) & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix}$$

Composite Transformations of Different Kinds

- Composite transformation of different kinds or simply called Concatenation of Transformations.
- Example: Derive the matrix representation of the General Pivot-Point Rotation.
- With a graphics package that only provides a rotate function for revolving objects about the coordinate origin, we can generate rotations about any selected pivot point (xc yc) by performing the following sequence of translate-rotate-translate operations:

Composite Transformations of Different Kinds

1. Translate the object so that the pivot-point position is moved to the coordinate origin.
2. Rotate the object about the coordinate origin.
3. Translate the object so that the pivot point is returned to its original position.

- $$\begin{bmatrix} 1 & 0 & \boldsymbol{xc} \\ 0 & 1 & \boldsymbol{yc} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & \mathbf{0} \\ \sin\theta & \cos\theta & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\boldsymbol{xc} \\ 0 & 1 & -\boldsymbol{yc} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & \boldsymbol{xc}(\mathbf{1}-\cos\theta)+\boldsymbol{yc}\sin\theta \\ \sin\theta & \cos\theta & \boldsymbol{yc}(\mathbf{1}-\cos\theta)-\boldsymbol{xc}\sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

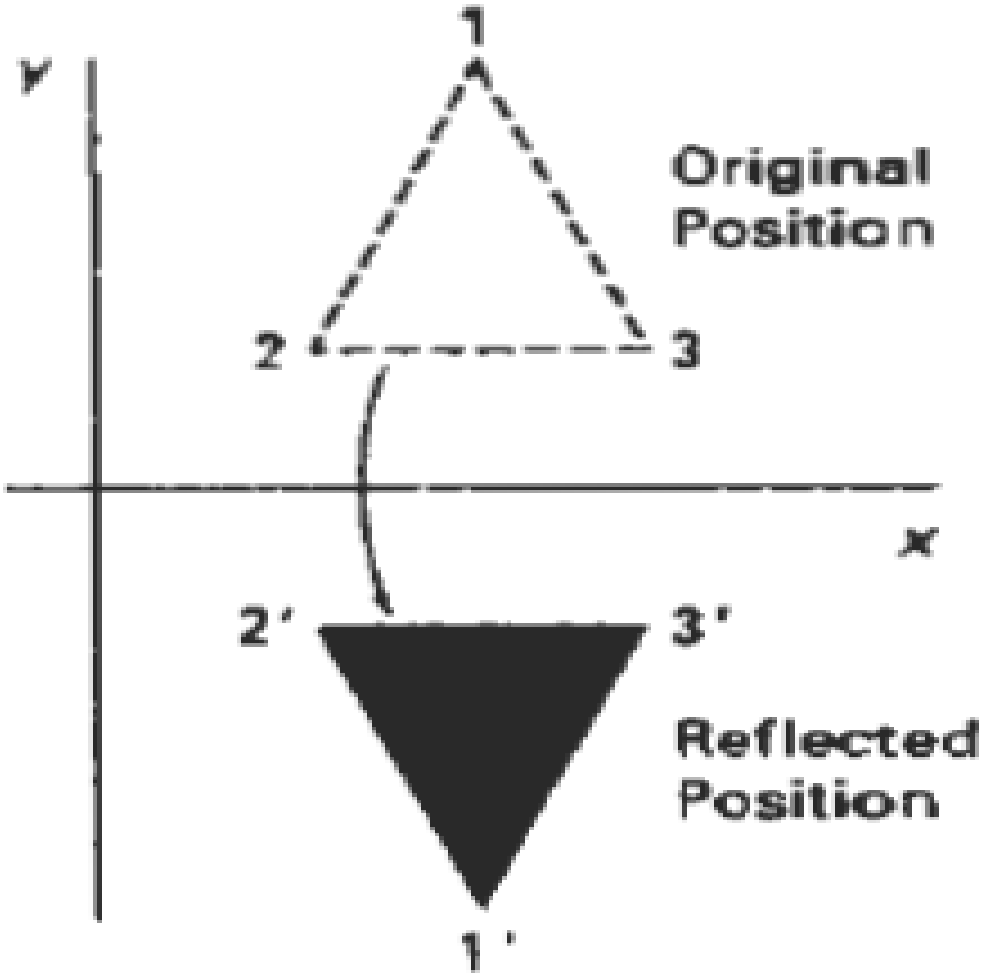
Reflection

- **A** reflection is a transformation that produces a mirror image of an object.
- The mirror image for a two-dimensional reflection is generated relative to **an axis of reflection** by rotating the object 180^0 about the reflection axis.
- We can choose an axis of reflection in the **xy** plane or perpendicular to the xy plane.
- When the reflection axis is a line in the xy plane, the rotation path about this axis is in a plane perpendicular to the xy plane.

- For reflection axes that are perpendicular to the xy plane, the rotation path is in the xy plane.
- Following are examples of some common reflections.
- **Reflection about the x-axis (line y = 0):**
- This reflection is accomplished with the transformation matrix:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{1} & 0 & \mathbf{0} \\ 0 & \mathbf{-1} & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- This transformation keeps ***x*** values the same, but "flips" the y values of coordinate positions.

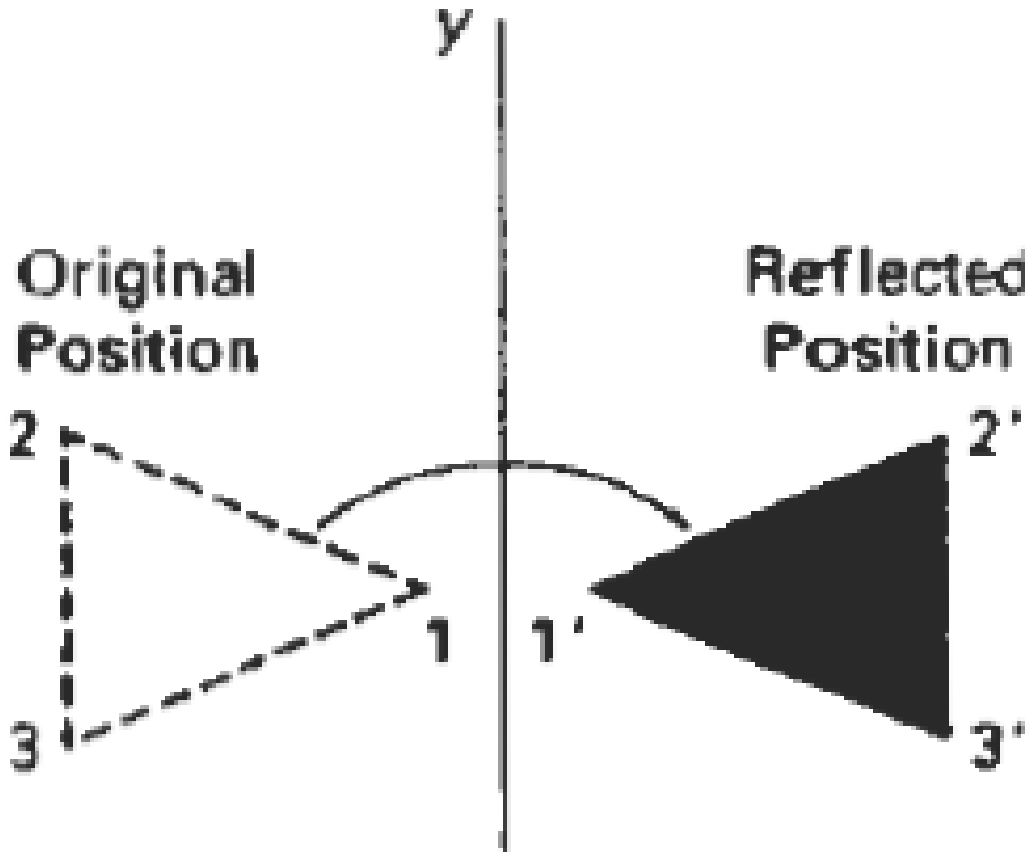


Reflection about the x-axis

- **Reflection about the y-axis (line x = 0):**
- This reflection is accomplished with the transformation matrix:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- This transformation keeps ***y*** values the same, but "flips" the x values of coordinate positions.



Reflection about the y-axis